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GCSE 9 -1
Mathematics
Higher Tier
Grade 9 'Tough Paper'
Paper 1

Total marks 80
1 Hour 30 minutes

PLEASE NOTE:
This paper does not claim the questions included are 'Grade 9 questions'.
This paper was designed for pupils aiming for Grade 9s who are looking for challenging questions within the GCSE 9-1 syllabus.

(1) Given that $x(a+bx)(a-bx) \equiv 25x - 4x^3$, find the value of b^{-a}.

(Total for Question 1 is 4 marks)

(2) Freda plays the lottery.

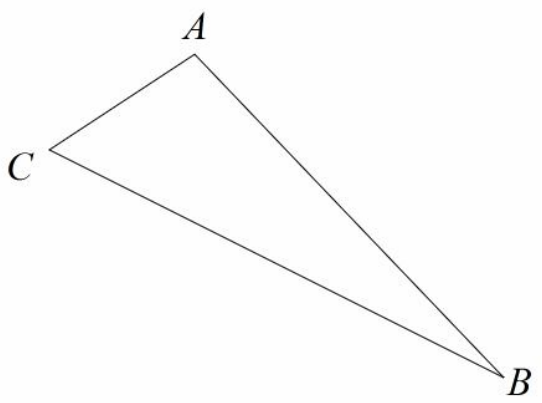
There are 49 balls to choose from.
The balls are numbered 1 – 49.

Freda chooses the 6 numbers shown below in the order in which they appear.

John believes the numbers were chosen randomly.
Show that John could be wrong stating a reason for your choice.

(Total for Question 2 is 4 marks)

(3) Triangle ABC is shown in the diagram below.



$AC = x$
$BC = 3x$
Angle $ACB = 60^o$

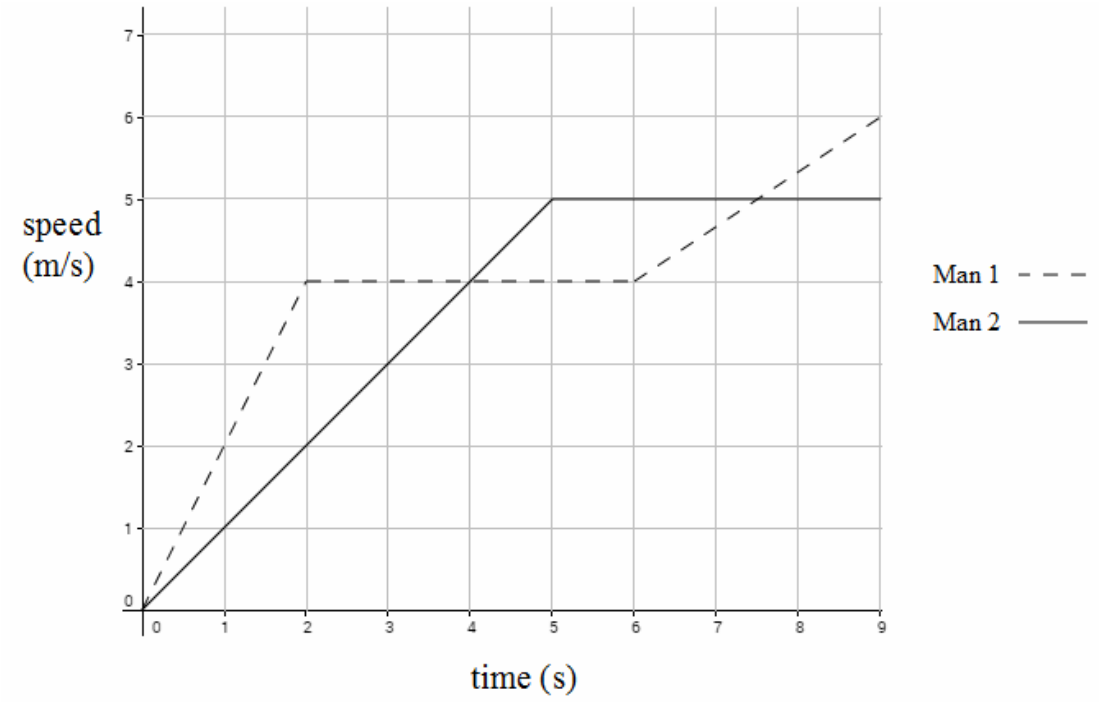
Show that the perimeter of the triangle is $\left(4+\sqrt{7}\right)x$.

(Total for Question 3 is 5 marks)

(4) Find the value of $\left(\frac{1}{0.16}\right)^{1.5}$

(Total for Question 5 is 2 marks)

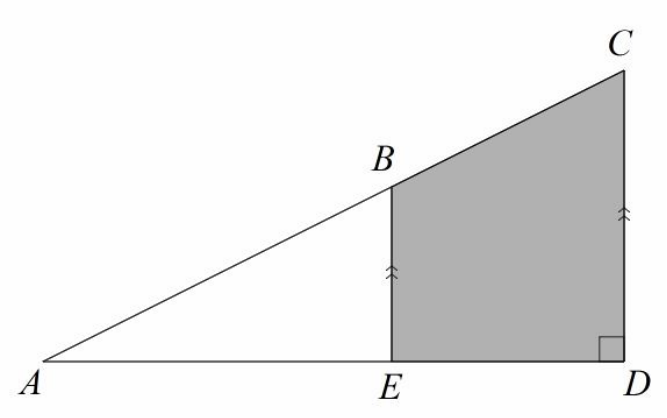
(5) Two men walk together along a road, starting at the same time.
The speed-time graph below shows the first 9 seconds of the walk.



The ratio of the distance covered by Man 1 to the distance covered by Man 2 in the first 9 seconds of the walk can be written in the form $m:n$ where m and n are double digit integers.
Find the value of m and the value of n.

(Total for Question 5 is 5 marks)

(6) Triangle ACD is shown in the diagram below.



AED is a straight line.

$AB = 3\sqrt{5}$

$AE = 2BE$

$3AD = 5AE$

Find the area of the shaded quadrilateral $BCDE$.

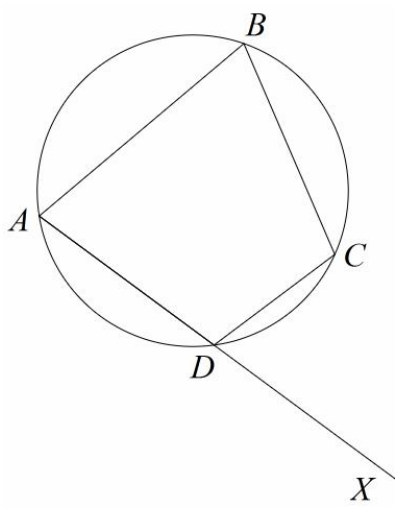
(Total for Question 6 is 5 marks)

(7) Find the value of p :

$$\left(\frac{\cos(60^o)}{\sin(60^o)}+\frac{10}{\sqrt{12}}\right)^2 = p$$

(Total for Question 7 is 4 marks)

(8) A, B, C and D are all points on the circumference of a circle as shown in the diagram below.



Angle $DAB = x^2 - 5x - 8$
Angle $BCD = x^2 + 4x - 88$
Angle $CDA = y^2 - 15y + 90$
Angle $ABC = 5y - 6$

A line is drawn from D to X .
Angle $CDX = x^2 - 70$

Prove that ADX is a straight line.

(Total for Question 8 is 6 marks)

(9) The first five terms of an arithmetic sequence are:

$$x+1,\quad 2x,\quad \frac{2(2x+3)}{6-x},\quad x^2-2,\quad 5x-3$$

Show that the term $4x^2-3$ is not in the sequence.

(Total for Question 9 is 6 marks)

(10) A solid hemispherical piece of gold with diameter $1.2\times10^{2}\,cm$ is to be melted into identical solid cuboids.
The dimensions of the cuboids are $30cm, 12cm$ and $10\pi cm$.
Find the number of cuboids that can be made from the hemisphere.

(You may assume there is no wastage in the production process.)

(Total for Question 10 is 5 marks)

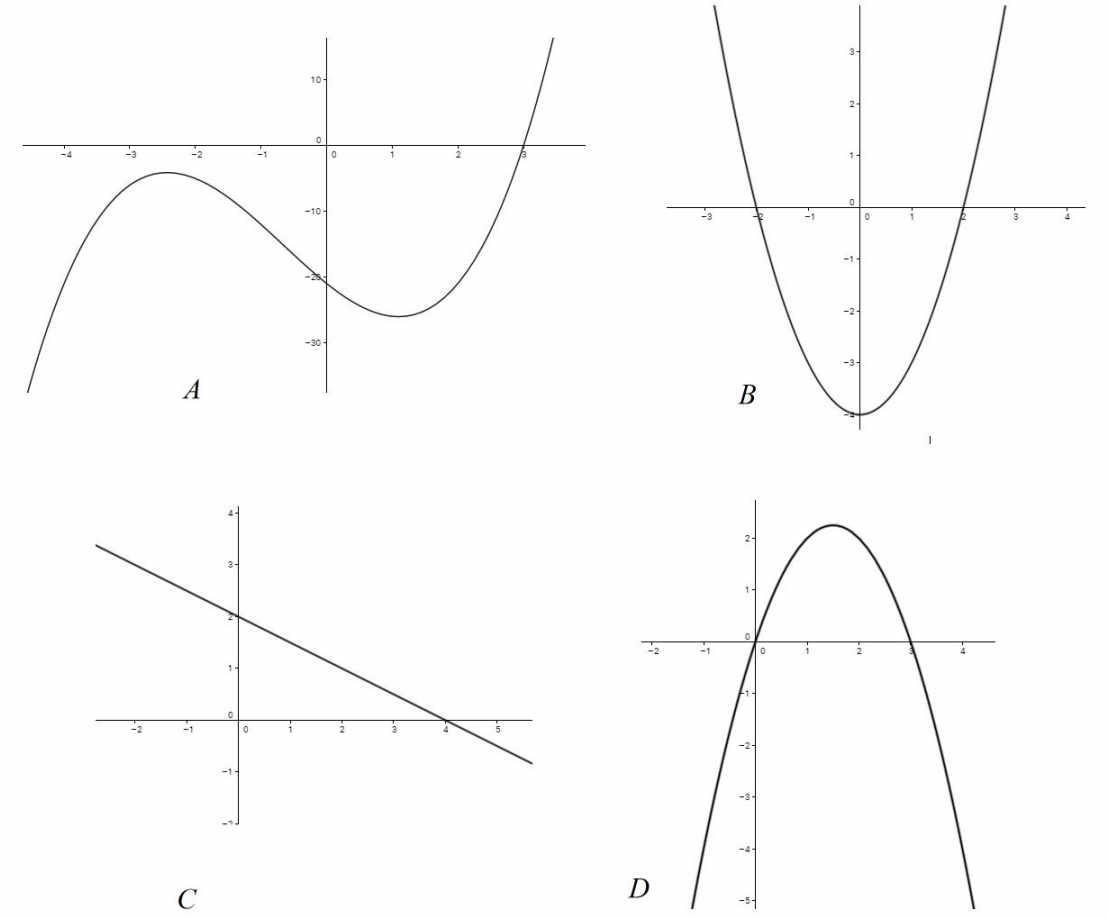
(11) Two functions are given below:

$$f(x) = (x + p)(x + q)$$

$$g(x) = \frac{r}{x}, \quad x \neq 0$$

p, q and r are constants.

State which of the following graphs could be used to solve the equation $f(x) = g(x)$
You must give a reason for your choice.



(Total for Question 11 is 3 marks)

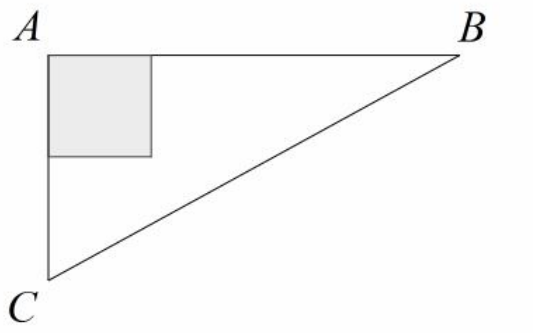
(12) A is inversely proportion to $B^{\frac{1}{3}}$ and C is directly proportional to the square of B.
When $A = 0.5$, $B = 64$.
When $C = 15$, $B = 5$.
Express C in terms of A.

(Total for Question 12 is 5 marks)

(13) Mr Lucky plays two games.
The two games are Game A and Game B.
Playing Game A and playing Game B are independent events.
The probability that Mr Lucky wins both games is $\frac{9}{25}$.
The probability that Mr Lucky wins Game B is four times greater than the probability of him losing Game A.
Find the probability that Mr Lucky wins only one of the two games he plays.
You must show full workings.

(Total for Question 13 is 6 marks)

(14) The diagram below shows the triangle ABC with a shaded square drawn inside.

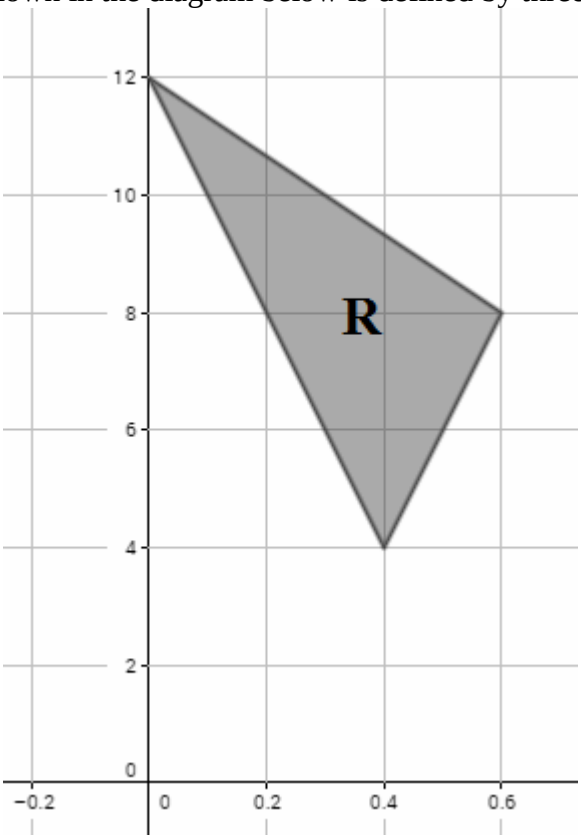


Triangle ABC is half of an equilateral triangle.
Angle $BAC = 90^o$
The shaded square touches the lines AB and AC and has side length $0.25AB$.

Show that the ratio of the area of the triangle to the area of the square is $8 : \sqrt{3}$

(Total for Question 14 is 5 marks)

(15) The region **R** shown in the diagram below is defined by three inequalities.



Find the three inequalities in the form $ay + bx \geq c$ where a, b and c are integers.

(Total for Question 15 is 6 marks)

(16) Two vectors are defined as follows:

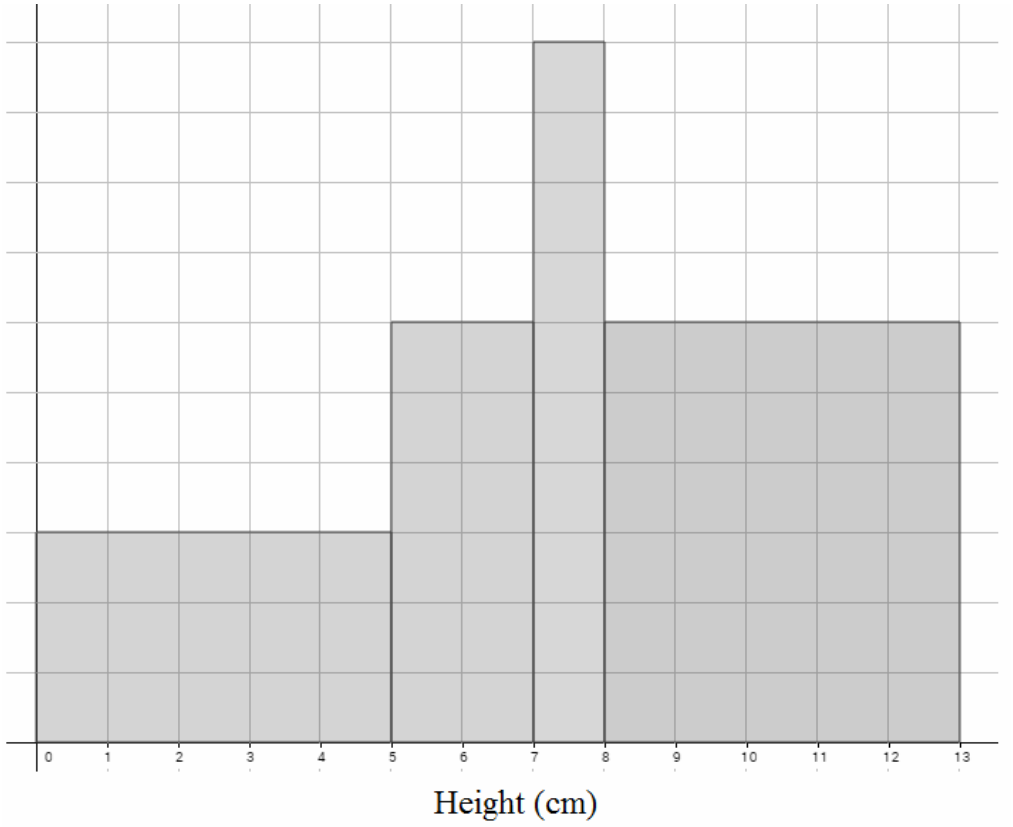
$$\overrightarrow{AC} = \begin{pmatrix} -2 \\ -1 \end{pmatrix}$$

$$\overrightarrow{AB} = \begin{pmatrix} -1 \\ -4 \end{pmatrix}$$

Find the value of $\cos(ACB)$ in its simplest form.

(Total for Question 15 is 5 marks)

(17) The histogram below shows information about the height (cm) of a number of plants.



There were 40 plants between 7 and 8cm tall.

Michael takes two plants at random from the sample and doesn't replace them.
He writes down his calculations for the probability and its answer as:

$$\frac{30}{67}\times\frac{16}{89}=\frac{480}{5963}.$$

Write down the minimum height of each of the plants Michael chooses.

(Total for Question 17 is 5 marks)
TOTAL FOR PAPER IS 80 MARKS